Example. A slabless stair has seven treads, each 300 mm wide with risers 180 mm high, the thickness of both being 100 mm. It is to carry an imposed load of 3 kN/m^2 and is 1 metre wide. Design the stair.

The self-weight of the treads and risers (assuming no finishes are required) is given by

$$\frac{(0.3 \times 0.1) + (0.18 \times 0.1)}{0.3} \times 24 = 3.84\,\text{kN/m}^2$$

Thus total ultimate load = $[(3.84 \times 1.4) + (3.0 \times 1.6)]\ 1.0 = 10.18$ kN/m. Since $l_t = 300$ mm, $l_r = 180$ mm and $h_t = h_r = 100$ mm, $k = 180/300 = 0.6$. From the chart on *Table 176*, the support-moment coefficient = -0.088. Thus

$$\text{support moment} = -0.088 \times 10.18 \times 2.1^2 = -3.95\,\text{kN m}$$

If $j = 7$,

$$\text{free bending moment} = \tfrac{1}{8} \times 10.18 \times 2.1^2\left(\frac{7^2+1}{7^2}\right)$$
$$= 5.73\,\text{kN m}$$

Therefore maximum bending moment at midspan = $5.73 - 3.95 = 1.78$ kN m.

25.2.4 Helical stairs

By using strain-energy principles it is possible to formulate, for symmetrically loaded helical stairs with fixed supports, the following two equations in M_0 and H:

$$\begin{aligned}M_0&[K_1(k_5 + \tfrac{1}{2}\sin 2\theta) + k_5k_7]\\ &+ HR_2[-K_1k_4 \tan\phi + k_4k_7 \tan\phi + k_5 \sin\phi\cos\phi(1-K_2)]\\ &+ nR_1^2[K_1(k_5 + \tfrac{1}{2}\sin 2\theta - \sin\theta) + k_5k_7 + k_6k_7R_2/R_1] = 0\end{aligned}$$

$$\begin{aligned}M_0&[-K_1k_4 + k_4k_7 + (k_7 - K_2)k_5]\\ &+ HR_2[\tfrac{1}{2}K_1 \tan\phi(\tfrac{1}{3}\theta^3 - \tfrac{1}{2}\theta^2 \sin 2\theta - 2k_4)\\ &+ \tfrac{1}{2}k_7 \tan\phi(\tfrac{1}{3}\theta^3 + \tfrac{1}{2}\theta^2\sin 2\theta + 2k_4) + 2k_4\tan\phi(k_7 - K_2)\\ &+ k_5\cos^2\phi(\tan\phi + K_2\cot\phi)] + nR_1^2[K_1(k_6 - k_4) + k_4k_7\\ &+ k_7(\theta^2\sin\theta + 2k_6)R_2/R_1 + (k_7 - K_2)(k_5 + k_6R_2/R_1)] = 0\end{aligned}$$

where

$$\begin{aligned}k_4 &= \tfrac{1}{4}\theta\cos 2\theta - \tfrac{1}{8}\sin 2\theta\\ k_5 &= \tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\\ k_6 &= \theta\cos\theta - \sin\theta\\ k_7 &= \cos^2\phi + K_2\sin^2\phi\end{aligned}$$

$K_1 = GC/EI_1$, $K_2 = GC/EI_2$ and $\theta = \beta/2$.

These simultaneous equations may be solved on a programmable calculator or larger machine to give coefficients k_1 and k_2 representing M_0 and H respectively. If the values of M_0 and H are then resubstituted into the equations given on *Table 176*, the bending and torsional moments, shearing forces and thrust at any point along the stair may be calculated rapidly. Since the critical quantity controlling helical stair design is normally the vertical moment M_{vs} at the supports, a further coefficient k_3 can be derived to give this moment.

In ref. 127, Santathadaporn and Cusens present 36 design charts for helical stairs, covering ranges of β of 60° to 720°, ϕ of 20° to 50°, b/h of 0.5 to 16 and R_1/R_2 of 1.0 to 1.1, based on a ratio of G/E of 3/7. The four design charts provided on *Table 177* have been recalculated for a ratio of G/E of 0.4 as recommended in CP110 and by taking C to be one-half of the St Venant value for plain concrete. These charts cover ranges of β of 30° to 360° and ϕ of 20° to 40°, with values of b/h of 5 and 10 and R_1/R_2 of 1.0 and 1.1, these being the ranges most frequently met in helical stair design. Interpolation between the various curves and charts on *Table 177* will be sufficiently accurate for preliminary design purposes.

Example. Design a helicoidal stair having an angle of inclination ϕ of 25° to the horizontal plane to support a uniform imposed load of 3 kN/m^2. The stair is to have a width of 1.2 m and the minimum thickness of the slab is 120 mm, the radius to the inside of the stair R_i being 900 mm. The angle β turned through by the stair is 240°.

For radius of the centre-line of the load R_1 is

$$R_1 = \frac{2(R_0^3 - R_i^3)}{3(R_0^2 - R_i^2)} = \frac{2(2.1^3 - 0.9^3)}{3(2.1^2 - 0.9^2)} \simeq 1.58\,\text{m}$$

Then since the radius of the centre-line of the stair R_2 is $0.9 + (1/2)1.2 = 1.5$ m, $R_1/R_2 \simeq 1.05$ and $b/h = 1200/120 = 10$. Thus from the charts on *Table 177*, interpolating as necessary, $k_1 = -0.12$, $k_2 = +1.52$ and $k_3 = -0.32$. Assuming that the mean thickness in plan of the stair (including treads and finishes) is 220 mm, the self-weight of the stair is $0.22 \times 24 = 5.3$ kN/m^2 and thus the total ultimate load = $(3 \times 1.6) + (5.3 \times 1.4) = 12.22$ kN/m^2. Thus

$$M_0 = -0.12 \times 12.22 \times 1.5^2 \times 1.2 = -3.96\,\text{kN m}$$
$$H = 1.52 \times 12.22 \times 1.5 \times 1.2 = 33.4\,\text{kN}$$

Also

$$M_{vs} = -0.32 \times 12.22 \times 1.5^2 \times 1.2 = -10.56\,\text{kN m}$$

The slab should now be checked to ensure that the thickness provided is sufficient to resist this final moment. Then, assuming this is so, the foregoing values of M_0 and H can be substituted into the equations for M_v, M_n, T, F, V_n and V_h given on *Table 176* to obtain the moments and forces along the stair, in order to detail the reinforcement. For example, when $\theta = 60°$, $M_v = 1.11$ kN m, $M_n = -48.17$ kN m, $T = 0.05$ kN m, $N = -36.5$ kN, $V_n = 9.68$ kN and $V_h = 16.7$ kN.

Typical distributions of moments and forces along the stair are shown on *Table 176*.

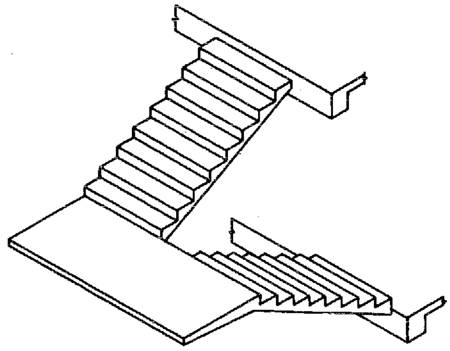
25.3 NON-PLANAR ROOFS

25.3.1 Prismatic structures

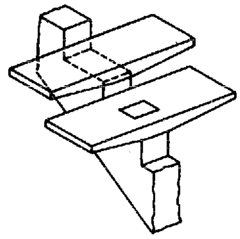
To design a simple prismatic roof or similar structure comprising a number of planar slabs for service loads and stresses, the resultant loads Q acting perpendicularly to each slab and the unbalanced thrusts N acting in the plane of each slab are determined first, taking into account the thrust of one slab on another. The slabs are then designed to resist the transverse bending moments due to loads Q assuming continuity and combination with the thrusts N. The longitudinal forces F due to the slabs bending in their own plane

Stairs 1: general information

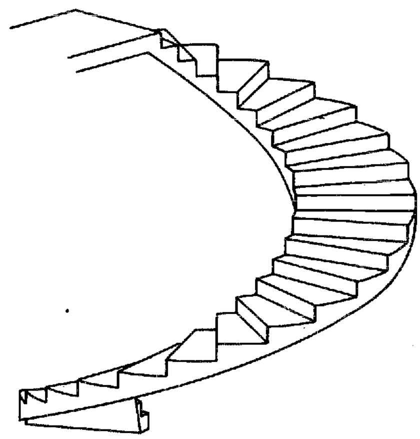
Free-standing (or scissor) stair (landing unsupported)

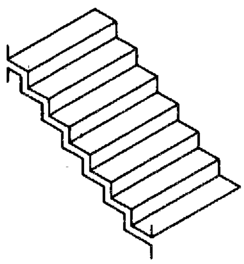


Individual precast treads cantilevered from spine beam

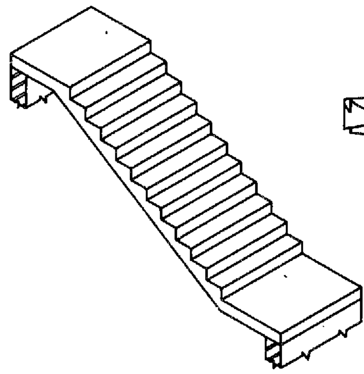


Helical stair

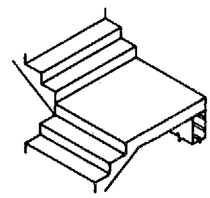




Slabless (or sawtooth or 'dog-leg') stair



Simple straight stair



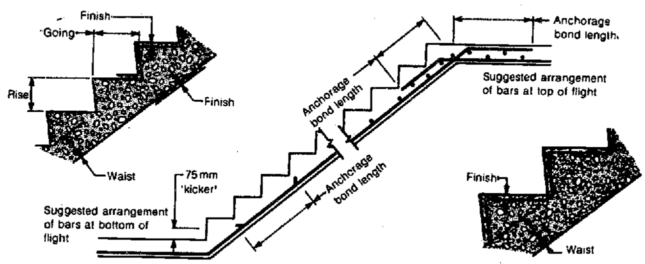
Landing arrangement for simple stair

General information

Optimum dimensions for stairs (BS5395) (mm)

Usage	Going	Rise	Min. width
Public	300	150	1000
Semi-public	275	165	1000
Private	250	175	800

General optimum dimensions:
2 × rise + going = 600 mm



Free-standing stairs

If flights are freely supported at A and A′:

$$H=\left[n_l(b_1+b)\left(1+\frac{1}{2}\frac{b}{a}\sec\phi\right)+n_f a\cos\phi\right]\Big/2\tan\phi$$

If flights are fully fixed at A and A′:

$$H\simeq\left[n_l(b_1+b)\left(4+3\frac{b}{a}\sec\phi\right)+3n_f a\cos\phi\right]\Big/\left\{2\tan\phi\left(4+3(b_1/a)^2\Big/\left[\frac{0.72}{1+(h_f/b)^2}+\frac{1}{K}\right]\right)\right\}$$

$$M_0\simeq[Hb_1\tan\phi-\tfrac{1}{4}n_l(b_1^2-b^2)]\Big/\left[\frac{1.44K}{1+(h_f/b)^2}+2\right]$$

where $K=\left(\frac{h_f}{h_l}\right)^3\left(\frac{b_1}{a}\right)\sec^2\phi$

Then for OB, at any point distance y from O:

$$M_v=-M_0-\tfrac{1}{2}n_l y^2 \qquad M_h=-Hy \qquad T=-\tfrac{1}{2}n_l by$$

For BC, at any point distance y from O:

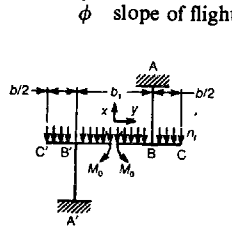
$$M_v=-\tfrac{1}{2}n_l[\tfrac{1}{2}(b_1+b)-y]^2 \qquad M_h=0$$
$$T=-\tfrac{1}{2}n_l b[\tfrac{1}{2}(b_1+b)-y]$$

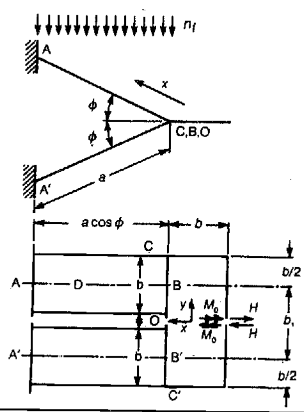
For AB, at any point distance x from B:

$$M_v=Hx\sin\phi-\tfrac{1}{2}n_l(b_1+b)(x\cos\phi+\tfrac{1}{2}b)-\tfrac{1}{2}n_f x^2\cos^2\phi$$
$$M_h=-\tfrac{1}{2}Hb_1\cos\phi-[M_0+\tfrac{1}{8}n_l(b_1^2-b^2)]\sin\phi$$
$$T=-\tfrac{1}{2}Hb_1\sin\phi+[M_0+\tfrac{1}{8}n_l(b_1^2-b^2)]\cos\phi$$

Additional notation

- a length of flight
- b width of flight and landing
- b_1 distance between centre-lines of flights
- H, M_0 horizontal restraint force and restraint moment at cut, respectively
- h_f, h_l slab depth of flight and of landing, respectively
- M_h, M_v, T horizontal and vertical bending moments and torsional moment at any point, respectively
- n_f, n_l ultimate load per unit length on flight and on landing, respectively
- x, y distances measured along flight and along Y-axis respectively
- ϕ slope of flight measured from horizontal





under the loads N are, for any two adjacent slabs AB and BC, calculated from formula (2) in *Table 178*, in which M_{AB} and M_{BC} are found from formula (1) if the structure is freely supported at the end of L. For each pair of slabs AB–BC, BC–CD etc. there is an equation like (2) containing three unknown forces F. If there are n pairs, there are $(n-1)$ equations and $(n+1)$ unknowns. The conditions at the outer edges a and z of the end slabs determine the forces F at these edges; for example, if the edges are unsupported, $F_a = F_z = 0$. The simultaneous equations are solved for the remaining unknown forces F_A, F_B, F_C etc. The longitudinal stress at any junction B is calculated from the formula (in *Table 178*) for f_B. Variation of the longitudinal stress from one function to the next is rectilinear. If f_B is negative, the stress is tensile and should be resisted by reinforcement. Shearing stresses are generally small.

25.3.2 Domes

A dome is designed for the total vertical load only, that is, for the weights of the slab and any covering on the slab, the weight of any ceiling or other distributed load suspended from the slab, and the imposed load. The intensity w of total service load = the equivalent load per unit area of surface of the dome. Horizontal service loads due to the wind and the effects of shrinking and changes in temperature are allowed for by assuming an ample imposed load, or by inserting more reinforcement than that required for the vertical load alone, or by designing for stresses well below the permissible values, or by combining any or all of these methods.

Segmental domes. Referring to the diagram and formulae in *Table 178*, the circumferential force acting in a horizontal plane in a unit strip S is T, and the corresponding force (the meridional thrust) acting tangentially to the surface of the dome is N. At the plane where θ is $51°48'$, that is, at the plane of rupture, $T = 0$. Above this plane T is compressive and reaches a maximum value of $0.5wr$ at the crown of the dome ($\theta = 0$). Below this plane T is tensile, and equals $0.167wr$ when $\theta = 60°$ and wr when $= 90°$. The meridional thrust N is $0.5wr$ at the crown, $0.618wr$ at the plane of rupture, $0.667wr$ when $\theta = 60°$, and wr when $\theta = 90°$; i.e. N increases from the crown towards the support and has its greatest value at the support.

For a concentrated load F on the crown of the dome, T is tensile; and T and the corresponding meridional compressive thrust N are given by the appropriate formulae in *Table 178*, the basis of which is that the load is concentrated on so small an area at the crown that it is equivalent to a point load. The theoretical stress at the crown is therefore infinite, but the practical impossibility of obtaining a point load invalidates the application of the formulae when $\theta = 0$ or very nearly so. For domes of varying thickness, see ref. 87.

Shallow segmental domes. Approximate analysis only is sufficient in the case of a shallow dome; appropriate formulae are given in *Table 178*.

Conical domes. In a conical dome, the circumferential forces are compressive throughout and at any horizontal plane x from the apex are given by the expression for T in *Table 178*, the corresponding force in the direction of the slope being N. The horizontal outward force per unit length of circumference at the bottom of the slope is T_r, and this force must be resisted by the supports or by a ring beam at the bottom of the slope.

25.3.3 Segmental shells

General notes on the design of cylindrical shell roofs and the use of *Table 179* are given in section 6.1.9.

Membrane action. Consideration of membrane action only gives the following membrane forces per unit width of slab due to the uniform loads shown on *Table 178*: to obtain stresses, divide by the thickness of shell h. Negative values of V indicate tension in the direction corresponding to an increase in x and a decrease in θ_x: positive values of F indicate tension. Reinforcement should be provided approximately in line with and to resist the principal tensile force. If the shell is supported along any edges the forces will be modified accordingly.

At any point:

Tangential force:

$$F_y = -(g + q\cos\theta_x)r\cos\theta_x$$

Longitudinal force:

$$F_x = -(1-x)\frac{x}{r}[g\cos\theta_x + 1.5q(\cos^2\theta_x - \sin^2\theta_x)]$$

Shearing force:

$$V_{xy} = (g + 1.5q\cos\theta_x)(2x - l)\sin\theta_x$$

Principal forces (due to membrane forces only):

$$F_p = \tfrac{1}{2}(F_x + F_y) \pm \sqrt{[\tfrac{1}{4}(F_x - F_y)^2 + V_{xy}^2]}$$

$$\tan 2\phi = \frac{2V_{xy}}{F_x - F_y}$$

At A (*at midpoint at edge: $\theta_x = \theta$; $x = l/2$*):

$$F_{yA} = -(g + q\cos\theta)r\cos\theta$$
$$F_{xA} = -\tfrac{1}{4}l^2[g\cos\theta + 1.5q(\cos^2\theta - \sin^2\theta)]/r$$
$$V_{xyA} = 0$$

At B (*midpoint at crown: $\theta_x = 0$; $x = l/2$*):

$$F_{yB} = -(g + q)r$$
$$F_{xB} = \tfrac{1}{4}l^2(g + 1.5q)/r$$
$$V_{xyB} = 0$$

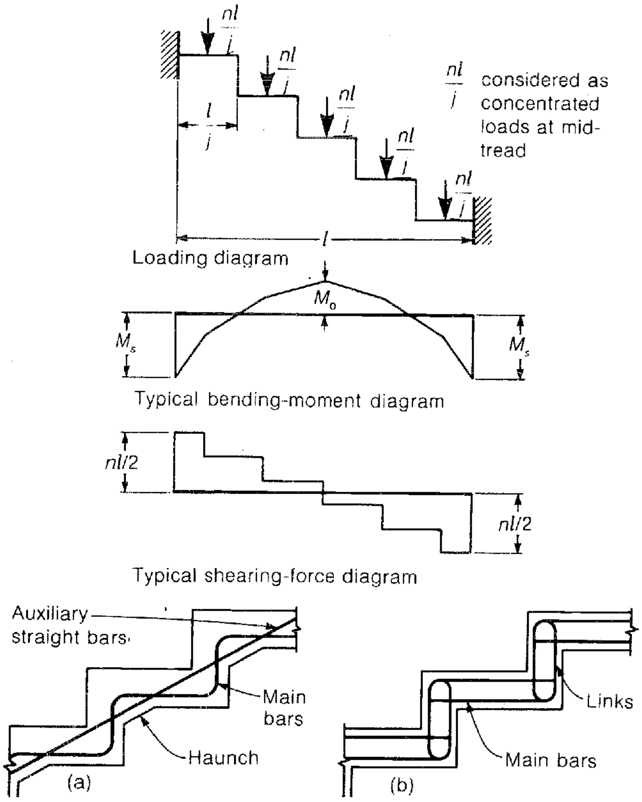
At C (*at support at edge: $\theta_x = \theta$; $x = 0$*):

$$F_{yC} = -(g + q\cos\theta)r\cos\theta$$
$$V_{xyC} = -(g + 1.5q\cos\theta)l\sin\theta$$
$$F_{xC} = 0$$

At D (*at support at crown: $\theta_x = 0$; $x = 0$*):

$$F_{yD} = -(g + q)r$$
$$F_{xD} = 0$$
$$V_{xyD} = 0$$

Sawtooth stairs

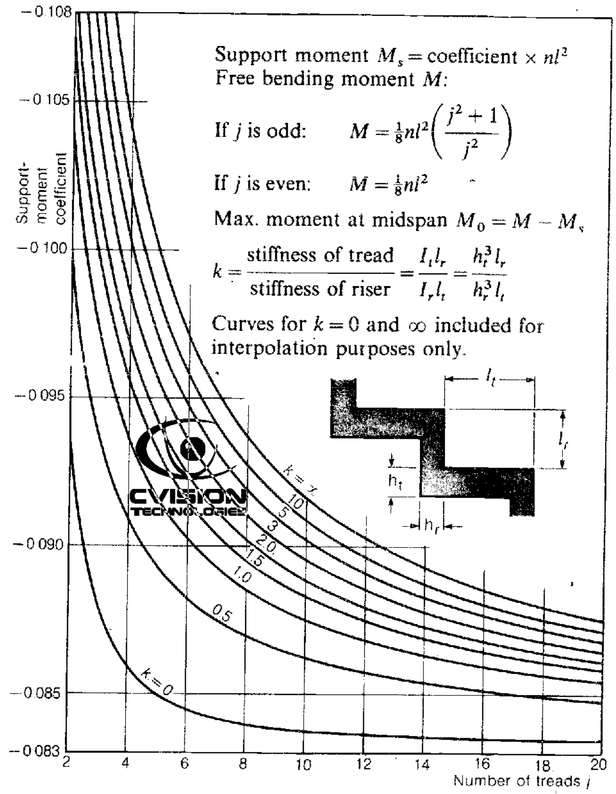


Loading diagram

Typical bending-moment diagram

Typical shearing-force diagram

Possible arrangements of reinforcement



Helical stairs

At any point along stair

Lateral moment: $M_n = M_0 \sin\theta \sin\phi - HR_2\theta \tan\phi \cos\theta \sin\phi - HR_2 \sin\theta \cos\phi + nR_1 \sin\phi (R_1 \sin\theta - R_2\theta)$

Torsional moment: $T = (M_0 \sin\theta - HR_2\theta \cos\theta \tan\phi + nR_1^2 \sin\theta - nR_1R_2\theta)\cos\phi + HR_2 \sin\theta \sin\phi$

Vertical moment: $M_v = M_0 \cos\theta + HR_2\theta \tan\phi \sin\theta - nR_1^2(1 - \cos\theta)$

Thrust: $N = -H \sin\theta \cos\phi - nR_1\theta \sin\phi$

Lateral shearing force across stair: $V_n = nR_1\theta \cos\phi - H \sin\theta \sin\phi$

Radial horizontal shearing force: $V_h = H \cos\theta$

where

Redundant moment acting tangentially at midspan: $M_0 = k_1 nR_2^2$

Horizontal redundant force at midspan: $H = k_2 nR_2$

Vertical moment at supports: $M_{vs} = k_3 nR_2^2$

Additional notation

I_1, I_2 second moment of area of stair section about horizontal axis and axis normal to slope, respectively

n total loading per unit length projected along centre-line of load

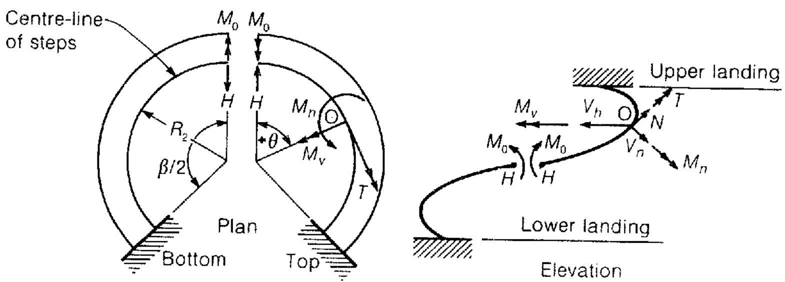
R_1 radius of centre-line of loading = $(2/3)(R_0^3 - R_i^3)/(R_0^2 - R_i^2)$

R_2 radius of centre-line of steps = $(1/2)(R_i + R_0)$, where R_i and R_0 are the internal and external radii of the stair, respectively

θ angle subtended in plan between point considered and midpoint of stair

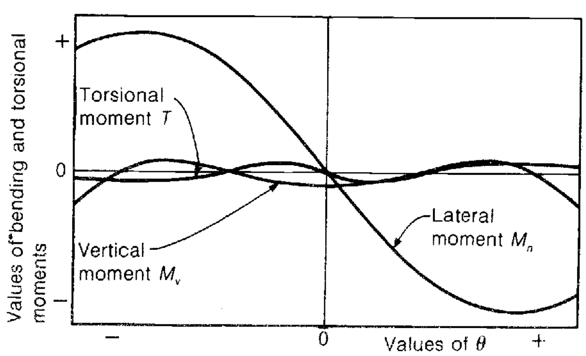
β total angle subtended by helix in plan

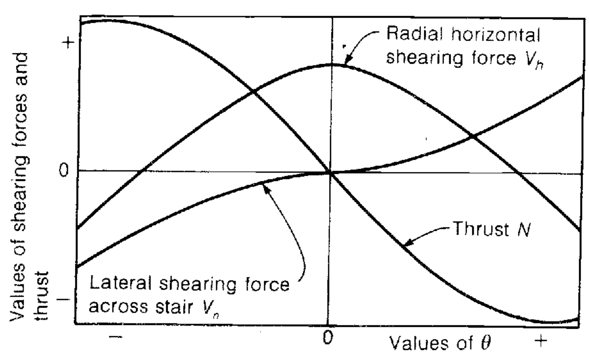
ϕ slope of tangent to helix centre-line measured from horizontal



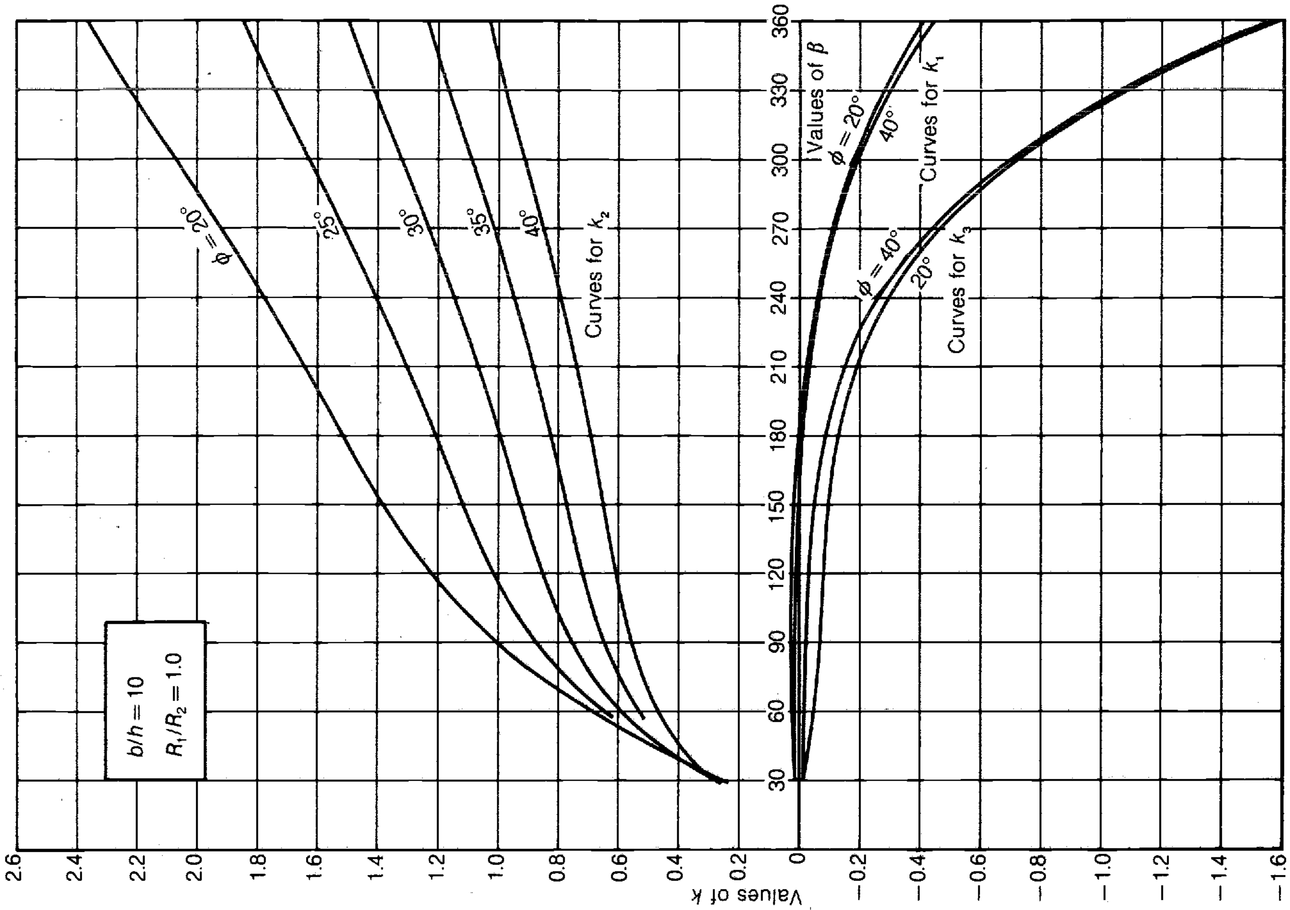
Plan

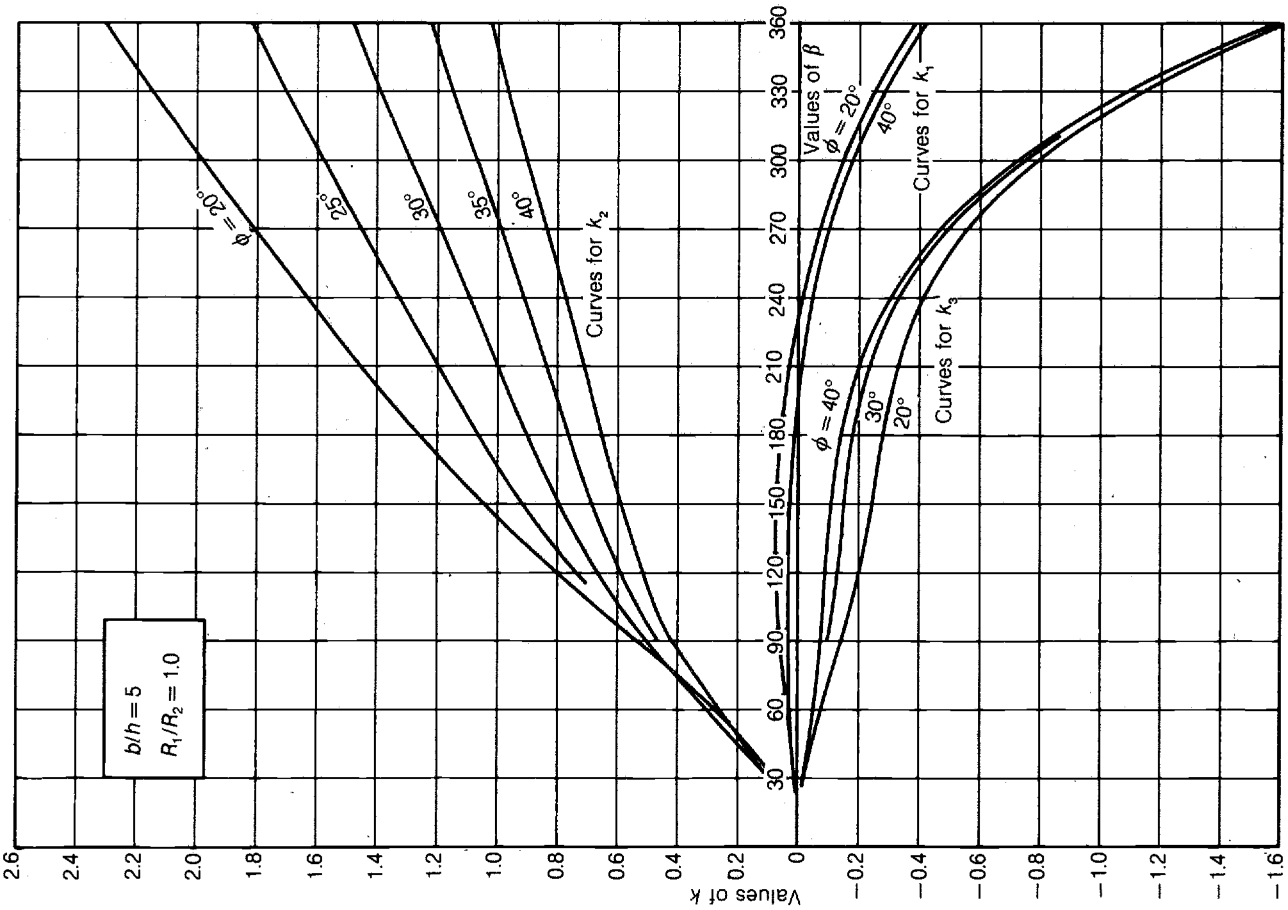
Elevation

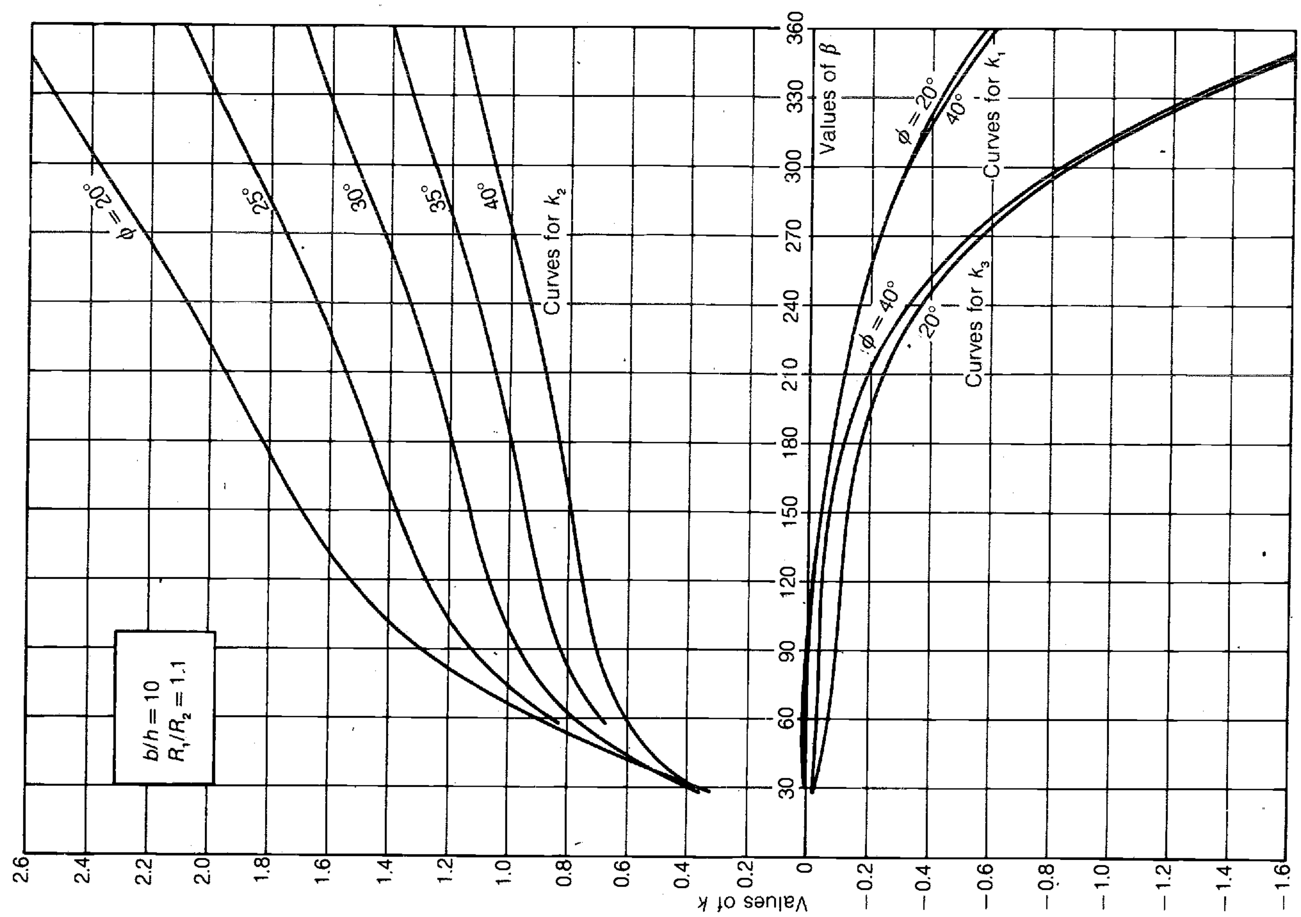


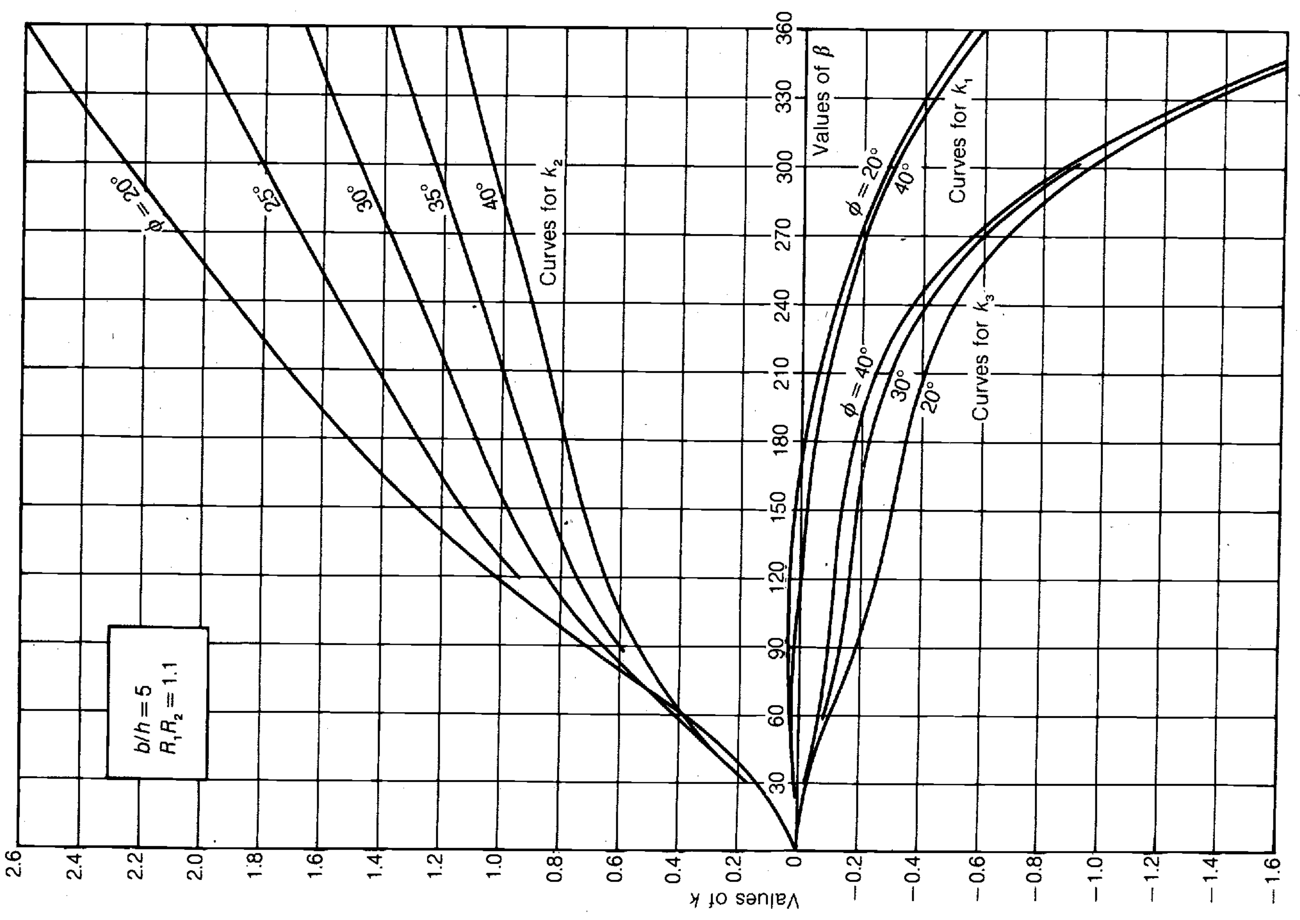


Stairs 3: design coefficients for helical stairs






b/h = 10
$R_1/R_2 = 1.1$
Curves for k_2
$\phi = 20°$
25°
30°
35°
40°
Values of β
Values of k
Curves for k_1
$\phi = 20°$
40°
Curves for k_3
$\phi = 40°$
20°


b/h = 5
$R_1R_2 = 1.1$
Curves for k_2
$\phi = 20°$
25°
30°
35°
40°
Values of β
Values of k
Curves for k_1
$\phi = 20°$
40°
Curves for k_3
$\phi = 40°$
30°
20°